A. Michael Noll is on the research staff of the Bell Telephone Laboratories in Murray Hill, New Jersey. He has worked in the field of two- and three-dimensional films and moving pictures generated by computer to display scientific information, and is interested in the computer as a creative medium in the arts. His computer-generated pictures have been exhibited at the Howard Wise Gallery, New York. As a logical extension of Mr. Noll's computer technique for generating two-dimensional pictures, it became possible to use computer-generated stereoscopic drawings to achieve a three-dimensional effect if viewed with polaroid glasses. From the three-dimensional still picture, the next logical step was to animate the picture and then to produce moving, three-dimensional objects or kinetic sculptures.

The use of computers in communication research has uncovered techniques which can be applied to other areas of the visual and performing arts. For example, the same technique may be used to simulate or record choreography for dance. New computer languages are being developed which will make it very easy for the artist to communicate instructions to the machine. Computers with display equipment will also become more readily accessible with the net result that many more artists and designers will become computer oriented.

COMPUTERS AND THE VISUAL ARTS

by A. Michael Noll

Opposite page: Enlarged segment of computer-produced picture.

The techniques and concepts which have been developed during the past few years for using computers to generate visual displays of scientific data can be applied with very little effort to the production of artistic visual displays. Previous experience has indicated that emotional feelings and prejudices often against the concept of computer art exist on the part of both artists and scientists. Actually, what artists may learn in using these new computer techniques could be valuable to scientists and engineers. Conversely, artists may discover that computers are very advantageous in many artistic endeavors. Thus, computers may play an important role in linking art and science, with the artist and scientist mutually assisting each other. But before all this can happen, the artist and scientist must become familiar enough with the other's field so that each can recognize the advantages of mutual cooperation.

Art has always depended upon science and technology to supply both the medium in which the work is done and the tools for doing it. The techniques are common whether the computer is used to generate visual displays of scientific data (e.g., shapes and motions of mechanical systems, mathematical rotations of n-dimensional objects, motions of atoms in a fluid) or shapes and motions which may be important in design or as an artistic medium.

COMPUTERS AND PICTURES

All digital computers are similar in that they manipulate binary numbers under the control of a stored set of instructions called a program. The actual manipulation is performed by electronic circuits and devices. Most computers consist of a memory, an arithmetic processor, and input and output equipment. The program is stored in the computer's memory and instructs the computer to accept input data, to perform arithmetical, logical, and organizational operations with this data, and finally to generate some form of output.

As an example, the stored program might first instruct the machine to read the numbers punched on a card into a portion of the memory allocated for data storage. The arithmetic processor might then fetch these numbers from the memory and add them all together. The final sum might be returned to memory and from there printed out by a typewriter. Each operation is controlled in sequence by the stored program.

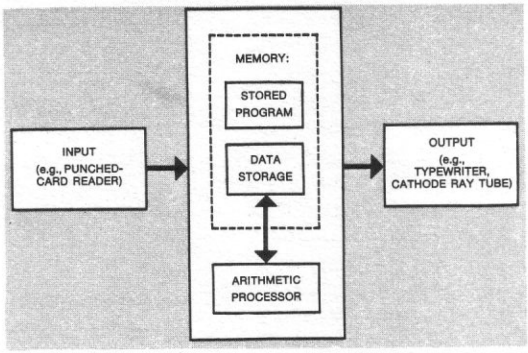
Computers are only capable of performing operations for which they have been programmed and instructed to perform. However, these machines are incomprehensibly fast in doing their tasks and are also capable of analyzing many factors to make different decisions. This extreme speed coupled with the inherent accuracy of digital calculation explains the desirability of computers.

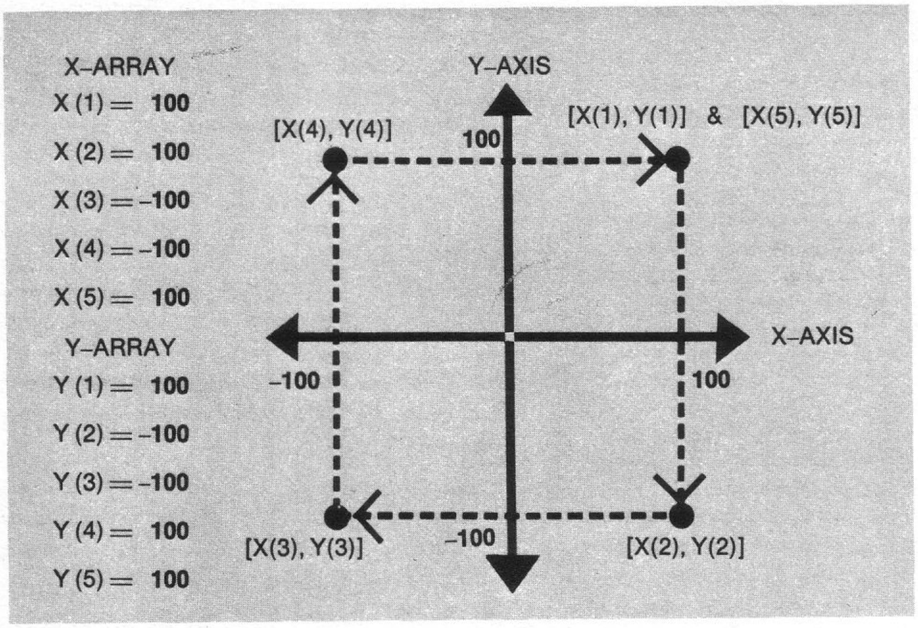
Recently, progress has been made in obtaining new output devices which allow the computer to produce visual output. One such device is the microfilm plotter.

The microfilm plotter consists of an interconnected cathode ray tube (conceptually similar to the picture tube in a television set), a camera, and some electronic control equipment. The cathode ray tube produces pictures on a phosphorescent screen with an electron beam which is electrically deflected across the screen to generate the desired picture. The camera photographs the face of the cathode ray tube. The required signals for deflecting the electron beam and for advancing the film come from the electronic control equipment, which includes circuitry for decoding instructions given to the microfilm plotter by the main computer. These instructions are commands for drawing straight lines between numerically specified points. In this way, pictures are generated on the microfilm plotter by instructions comprising a part of a program in the main computer.

As an example, suppose a square is to be generated on the microfilm plotter. The required program would first make two arrays of numbers whose elements are the X and Y coordinates of the corners of a square; i.e., $X(1) = 100$, $Y(1) = 100$, $X(2) = 100$, $Y(2) = -100$, $X(3) = -100$, $Y(3) = -100$, $X(4) = -100$, $Y(4) = 100$, $X(5) = 100$, $Y(5) = 100$. (The fifth point is required to close the square.) The program would, secondly, contain an instruction to advance the film to an un-

Diagram showing the basic concept for using the computer to produce typewritten or visual output.





Example of drawing a square with the microfilm plotter: Four points are located, each with a value of 100, in a quadrant of the X-Y coordinate system. An instruction to the microfilm plotter to connect these points would thus produce a square.

exposed frame. The next instruction would tell the microfilm plotter to draw a line between the five points whose coordinates are contained in the X and Y arrays. Thus, the point (X(1),Y(1)) would be connected to (X(2),Y(2)), (X(2),Y(2)) would be connected to (X(3),Y(3)), and so on. The last instruction would once again advance the film.

The arrays are blocks of sequential locations in the computer's memory that have been specified by the programmer as being associated with some name. For instance, in the above case the X and Y arrays each contain five storage locations where X(1) refers to the first location which contains the number 100.

In the preceding example of a square, the coordinates are all known and are therefore put into the X and Y arrays as constants. For more complicated pictures, the program might include instructions for mathematically calculating these coordinates. The essence of all this is that a picture can be specified numerically as a set of points which can be obtained as results of numeric calculations in a computer program. For complicated pictures, the only additional requirements would be more points and smaller line segments.

RANDOMNESS

A sequence of numbers would be described as random if an observer were unable to determine a formula for exactly predicting each number in the sequence. These numbers might represent the coordinates of points in a computer-generated picture. However, a practical picture cannot be infinite in size, and therefore some limit or range must be placed upon the set of permissible random numbers. For example, only random numbers between −67 and +240 might be allowed. Within a permissible set, certain numbers might occur more often than others, or alternatively the occurrence of all the numbers might be equally probable.

If all the random numbers in a sequence fall between the limits *a* and *b*, where *b* is greater than *a*, and if the occurrence of all these numbers is equally probable, then the numbers are said to have a uniform probability density. Such a sequence is specified by only the limits *a* and *b*. The occurrence of any number within these limits is just as probable as the occurrence of any other number.

Aside from the uniform probability density there is another density of considerable importance. It is called the normal or Gaussian density (after the mathematician Gauss who first formulated it mathematically). For a Gaussian sequence of random numbers, the numbers tend to cluster about an average. The larger the number compared with this average, the less probable is its occurrence. Sometimes the Gaussian sequence is "truncated" so that numbers much larger than the average are not allowed. The Gaussian density is also characterized by its standard deviation which is a measure of the spread of the random numbers about the average. For a very long sequence of Gaussian random numbers, 68.3 percent of the numbers fall within plus or minus one standard deviation of the average, 95.5 percent within two standard deviations, and 99.7 percent within three standard deviations.

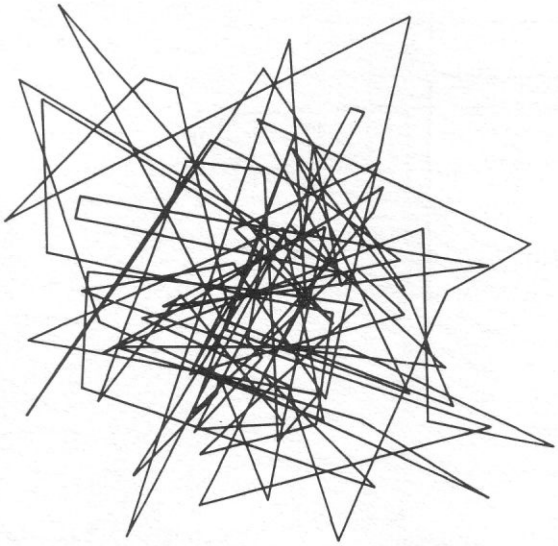
Numerical techniques developed for scientific and engineering use are available for generating a sequence of random numbers with either a uniform or Gaussian density. These techniques have been incorporated into special computer programs called subroutines which calculate the specified random sequence.

TWO-DIMENSIONAL PICTURES

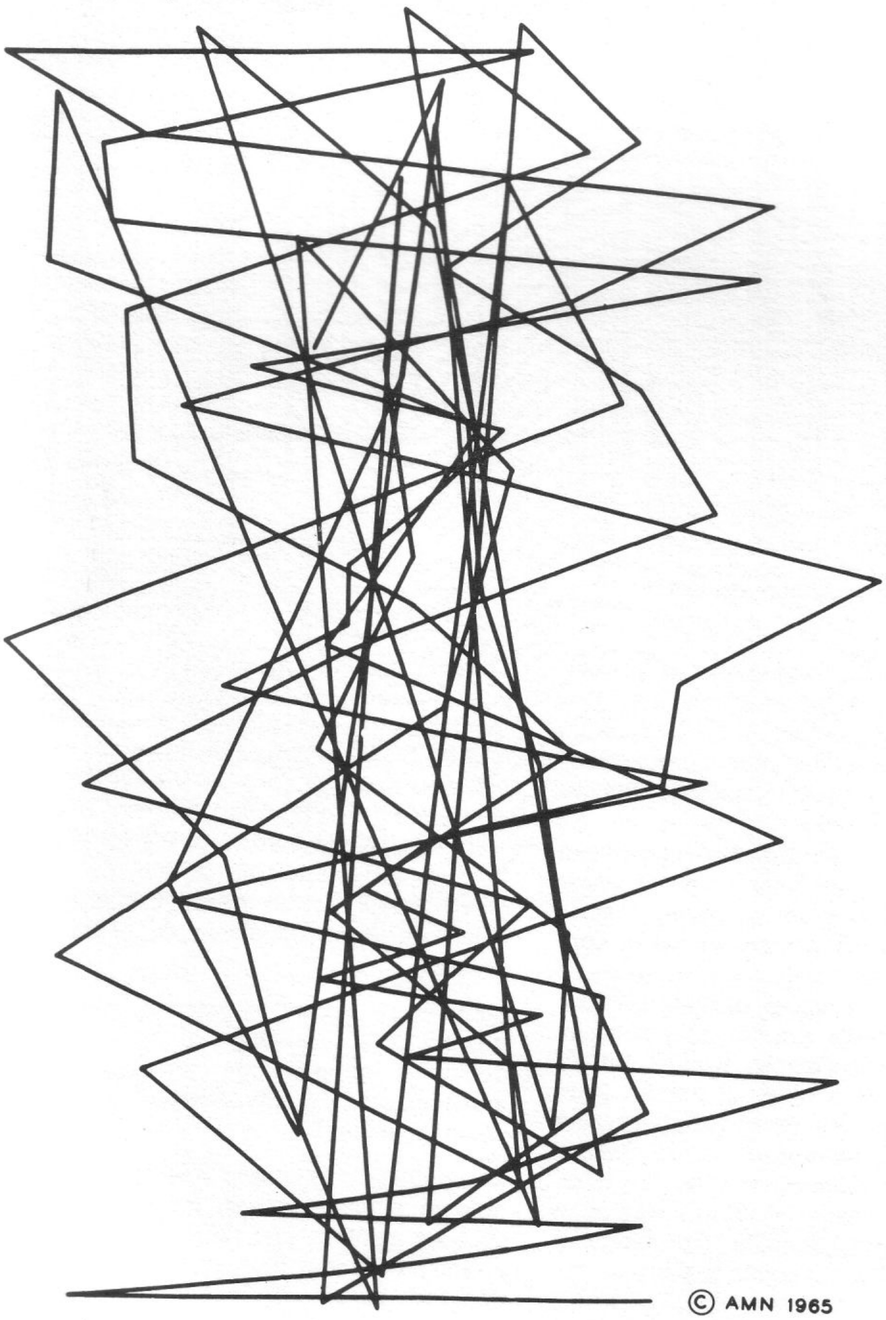
A computer program can be written instructing the machine to compute coordinates of points which, when connected together with straight lines, produce a picture.

Some interesting pictures can be generated with the computer by programming different random elements into the picture. In the first attempts along these lines, the random number subroutines for uniform and Gaussian densities

Picture with points determined by a random process and then connected by the microfilm plotter.



"Gaussian-Quadratic,"
© A. Michael Noll, 1965.



were used to generate arrays of numbers which became the coordinates of points. These points were then sequentially connected together with straight lines. The lines in the Gaussian picture cluster about the center since the average of the random coordinates was chosen to be the center of the picture. If desired, the computer could generate series of pictures in which the number of lines or the standard deviation was varied. The best picture could then be chosen from the series.

In general, completely random two-dimensional pictures are not very interesting. However, the computer is also able to mix together randomness and order in mathematically specified proportions to achieve a desired effect. The initial attempts at such mixing used Gaussian randomness for the X-axis coordinates but introduced a specified and non-random mathematical function for generating the Y-axis coördinates. A particularly good example of this mixing approach is shown in the picture, "Gaussian Quadratic". Ninety-nine lines join together 100 points whose horizontal positions are Gaussian. The vertical positions increase quadratically, i.e., the first point has a vertical position from the bottom of the picture given by $1^2 + 5x1$, the second point $2^2 + 5x2$, the third point $3^2 + 5x3$, etc. The maximum picture size is limited to 1024 units wide by 1024 units high, and thus the 30th point would be off the top of the picture ($30^2 + 5x30 = 1050$). To prevent this from happening, the vertical positions at the top are reflected to the bottom of the picture and then continue to rise. The result is a line that starts at the bottom of the picture and randomly zigzags to the top in continually increasing steps; at the top the line is "translated" to the bottom to continue its rise. The standard deviation of the Gaussian density is 150.

The exact proportions of "Gaussian Quadratic" were arrived at in a process of trial and error. The computer very rapidly produced series of pictures in which the different factors were uniformly changed. In this manner it became possible to bring an intuitive feeling for the pictorial effects of these factors into play.

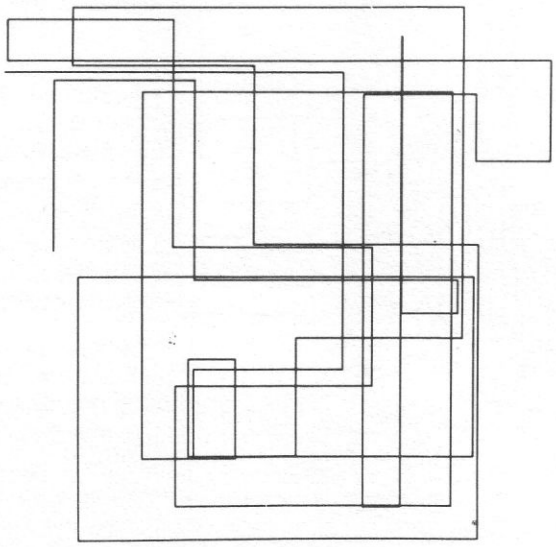
"Vertical-Horizontal" pictures were generated by a scheme in which only one of the two coordinates was changed (alternatingly) from one point to the next. The coordinates were otherwise random with a uniform probability density. "Vertical-Horizontal No. 1" consists of 50 lines with equal ranges in both directions; the number of lines in "Vertical-Horizontal No. 2" was increased to 300. "Vertical-Horizontal No. 3" consists of 100 lines with a range of -200 to $+200$ along the X-axis and a range of -500 to $+500$ along the Y-axis.

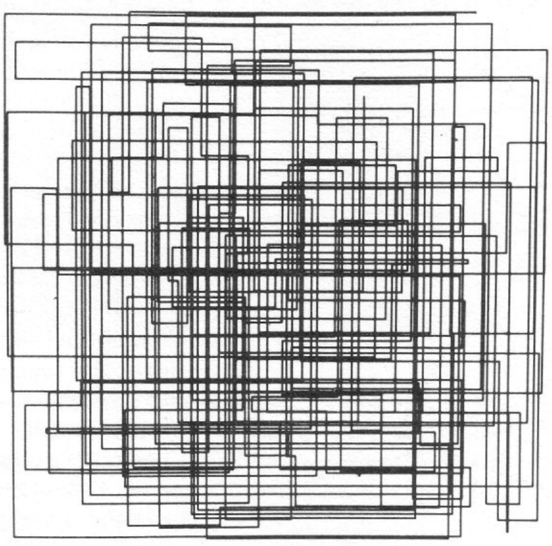
The preceding examples indicate that the computer in association with some method for obtaining visual output can be used to generate

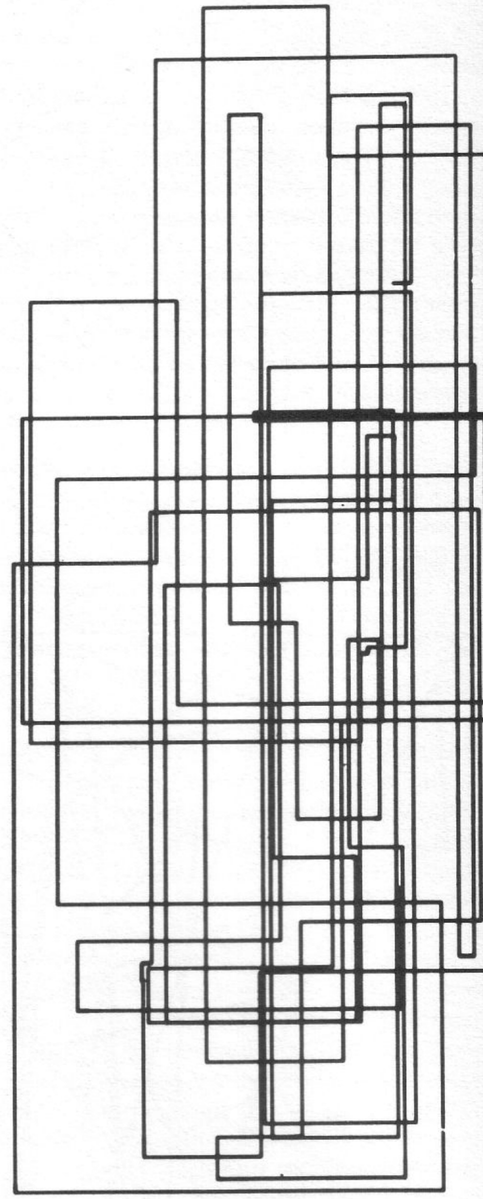
Computer-produced pictures by A. Michael Noll.

Right: "Vertical-Horizontal No. 1"

Bottom right: "Vertical-Horizontal No. 2"







Far right: "Vertical-Horizontal No. 3" © A. Michael Noll, 1965. In these pictures only one of the two coordinate values (X or Y) was changed from one point to the next along a continuous line. The change alternated between the X and the Y value. Otherwise the positions of the points were chosen at random with a uniform probability density.

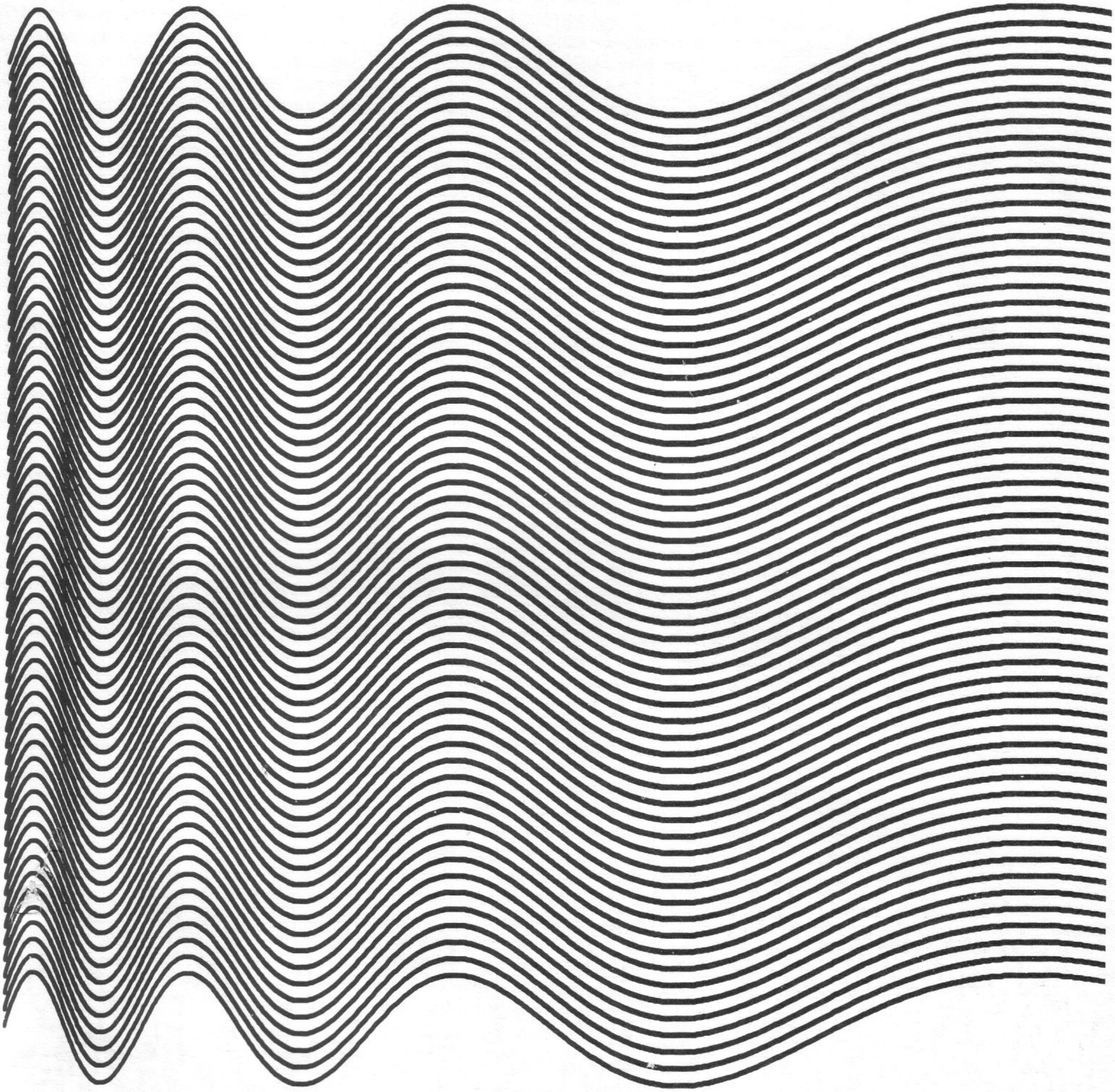
random and quasi-random abstract images. The artistic merit of the image thus generated is a matter of personal opinion, but it should be taken into account that the medium was in this case strongly limited by the exclusive application of black straight lines.

Many "op art" paintings are very regular and mathematical in design. The computer is extremely adept at constructing purely mathematical pictures and hence should be of considerable value to "op" artists. The drudgery of drawing or painting complex designs such as those in moiré patterns can be easily done by the machine. As an example, Bridget Riley's painting "Current" is a series of parallel lines that mathematically can be specified as sine waves with linearly increasing period. Such a formulation of her painting enables the computer to calculate an array of points based upon a simple mathematical formula. The microfilm plotter then connects the points to produce the finished result.

The fact that an "op art" picture can be adequately produced by a computer should not detract from the artistic merit of either the artist or the picture resulting from the artist-computer collaboration. The creative process takes place in the mind of the artist; the final painting is only the artist's rendition of his mental image. This is particularly true of "op art" although other types of art do involve the artist's interaction with his materials as part of the creative process.

Since most "op art" is definitely mathematical it is not at all surprising that the computer can duplicate "op" paintings. However, what could the computer do with different forms of abstract paintings?

This problem was approached by trying a quasi-random duplication with the computer of an abstract painting. The painting chosen for this investigation was Piet Mondrian's "Composition With Lines" (1917). This black and white



"Ninety computer-generated sinusoids with linearly increasing period." The top line of this picture was mathematically expressed as a sinusoid curve. The computer was then instructed to repeat the line 90 times. The result approximates closely Bridget Riley's painting "Current."

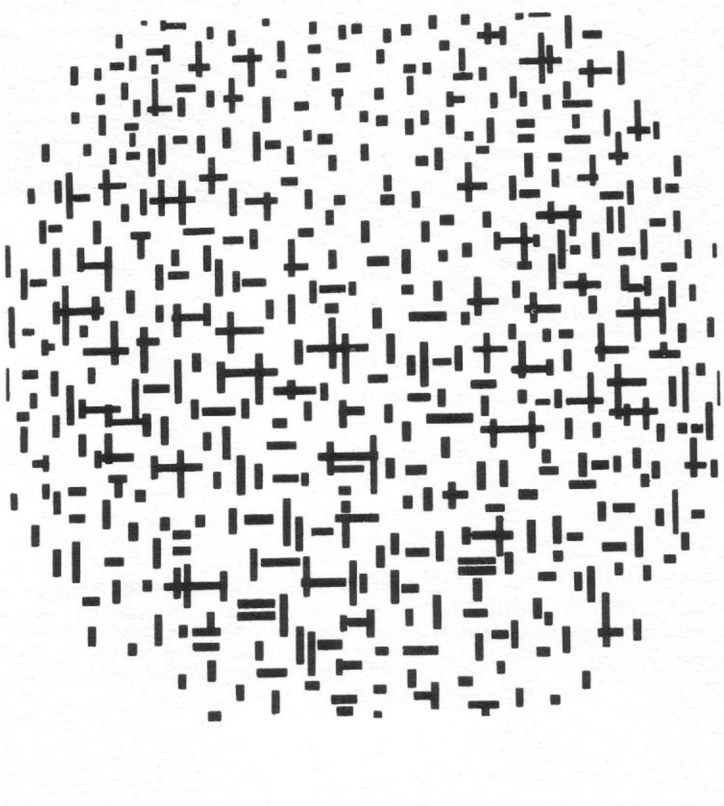
painting is from Mondrian's earlier period when he was experimenting with representations of the vertical and horizontal motifs of nature. "Composition With Lines" was chosen because it was composed entirely of solid black bars which could be drawn very easily by the microfilm plotter.

A cursory examination of Mondrian's painting reveals that, first, the outline of the painting is nearly circular except for cropping of the sides, top and bottom; second, the bars falling within a region in the upper half of the painting are shortened in length; third, the length and width of the bars otherwise appear random; and fourth, the placement of the bars is not random but seems to follow some scheme so that the entire space is almost uniformly covered.

A computer program was written utilizing the first three of the observations about Mondrian's painting. The placement of the bars was random with a uniform density. The vertical and horizontal bars were approximated as a series of closely spaced, and therefore overlapping, parallel line segments. These bars were placed randomly within a circle of radius 450 units. The choice of vertical or horizontal bars was equally likely. The width of the bars was equally probable between 7 and 10 units while the length of the bars was equally probable between 10 and 60 units. If the center of the bar fell inside a certain parabolic region with its origin at the center of the circle, the length of the bar was reduced by a factor proportional to the distance of the bar from the edge of the parabola. A trial-and-error approach ensured that the final picture was reasonably similar to the Mondrian painting.

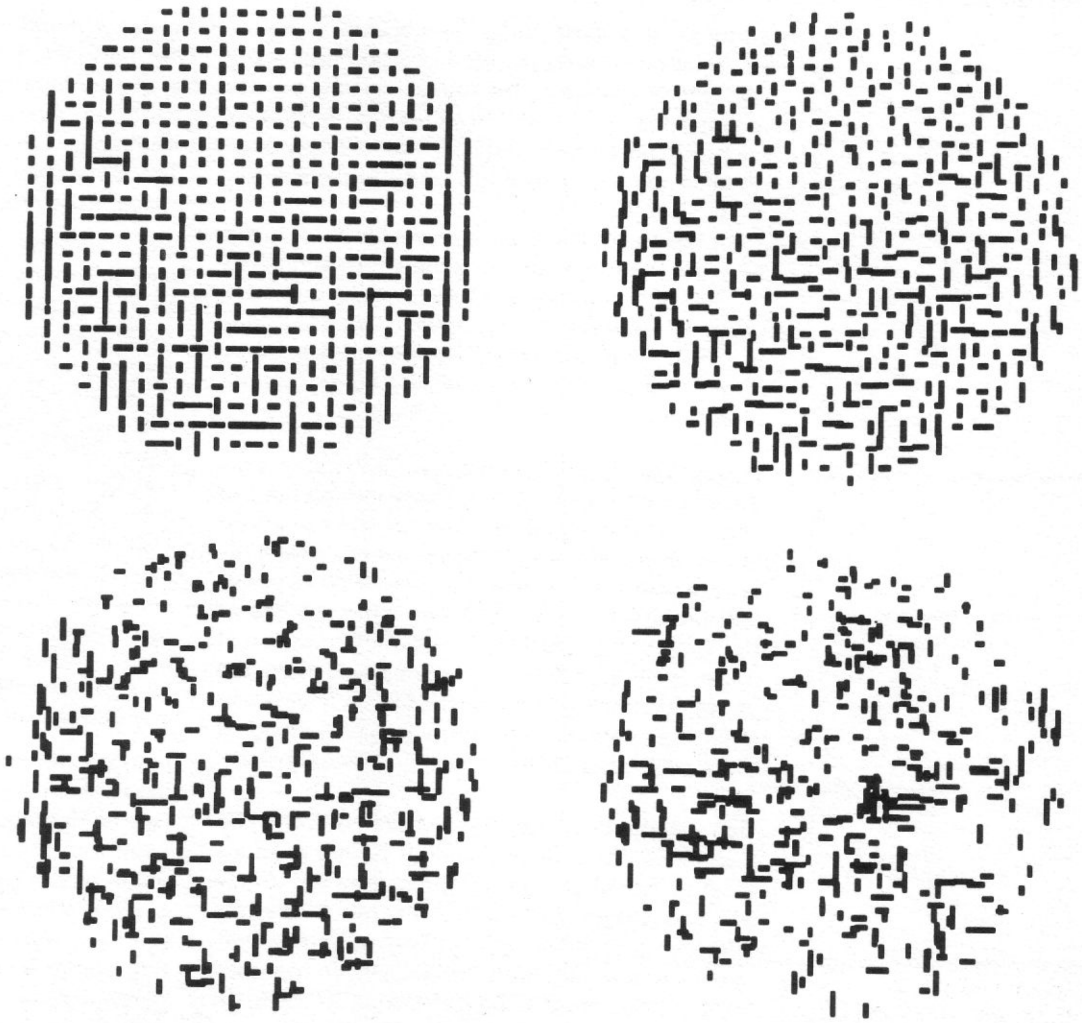
In a psychological experiment, a sample of one hundred artistically naive subjects were given reproductions of the Mondrian and of the computer picture.[1] These people were instructed to indicate which of the pictures they preferred and to identify the computer picture. In this particular group, 59% of the subjects preferred the computer picture and 28% identified correctly the computer pictures, i.e. 72% thought the Mondrian picture was done by computer. The sample group was composed of technical and clerical personnel who probably had no bias against computers. As indicated by questionary comments of these subjects, there was a trend among the non-technical people to associate randomness with creativity. Therefore, the results of this experiment neither discredit Mondrian nor imply that the computer is a greater artist than Mondrian, but raise the question to what extent randomness has aesthetical and emotional appeal. To answer this question, more experiments with controlled groups of subjects would be required.

The human-or-machine experiment, testing the preference of the two pictures, one computer-generated, the other by Mondrian, could raise a few more questions. For instance, although only one particular sequence of random numbers was used by the computer in producing this picture, it is not known if other pictures generated by statistically identical random sequences would be preferred over the Mondrian. The computer picture would clearly be more random than the Mondrian painting. However, more elaborate schemes could be used to produce pictures that even more closely resemble the Mondrian.



Bottom left: "Composition with Lines," (1917) by Piet Mondrian, © Rijksmuseum Kroller-Muller.

Bottom right: "Computer Composition With Lines," by A. Michael Noll in association with an IBM 7094 digital computer and General Dynamics SC-4020 microfilm plotter, © A. Michael Noll, 1965. This composition approximates Piet Mondrian's "Composition with Lines" (1917).



Four computer-generated random patterns based on the composition criteria of Mondrian's "Composition with Lines."

As a follow-up, another series of Mondrian-like computer pictures was generated. The scheme used to produce these pictures utilized random bar lengths and random bar widths within specified ranges. The bars were shortened if they fell within a parabolic region in the upper half of the picture. Only vertical bars were permitted along the sides of the picture. The actual positions of the bars were determined by adding a uniform-density random perturbation to an otherwise completely uniform grid-like set of positions. This random perturbation has a specified range; the range is zero and increases geometrically to a range of $\pm$ 250.

The conclusion of these investigations of computer-generated two-dimensional pictures is that the exciting potential of computers in art consists in their capability of producing mixtures of random elements with mathematically specified formulae for order. The experiments reported here involved only black and white pictures, but in the very near future color picture tubes will be controlled by computers, and infinitely variable color mixing will be possible. Presently, any artist desiring to use the computer would require a fairly sophisticated knowledge of computer programming. However, special "programming languages" that closely suit the needs of any particular artist could be developed, and these languages could be as natural to use as the conventional brushes and oils. Until recently the time lag between the running of the computer program and the finished picture has been several hours. Now, however, new display devices which immediately create an image on a large picture tube are being made available. Special lightpens are also available for writing and drawing on the face of the cathode ray tube and in this way it is possible to modify the picture. Such devices allow the computer-artist to sit at the console of the machine and to interact with the machine to produce a picture immediately. In the future it may even be possible for an artist to rent a console with a display device and work with a computer over distances. Many people would share the same mammoth facilities of a central utility computer. As leisure time increases and computer costs decrease, the use of time-shared computers for creative activities may become quite commonplace.

THREE-DIMENSIONAL PICTURES (COMPUTER SCULPTURE)

That three-dimensional pictures can be generated by the computer and a microfilm plotter by addition of a third dimension, is only a logical step further. This has already proved particularly valuable in depicting the motions of the basilar membranes in the ear in response to various sounds. The third dimension makes many phenomena far easier to understand and visualize.

The illusion of depth can be created if minutely different pictures are presented separately to the left and right eyes. These two pictures are obtained as perspective drawings of some object seen from two slightly different directions. This technique is formally called *stereographic projection*.

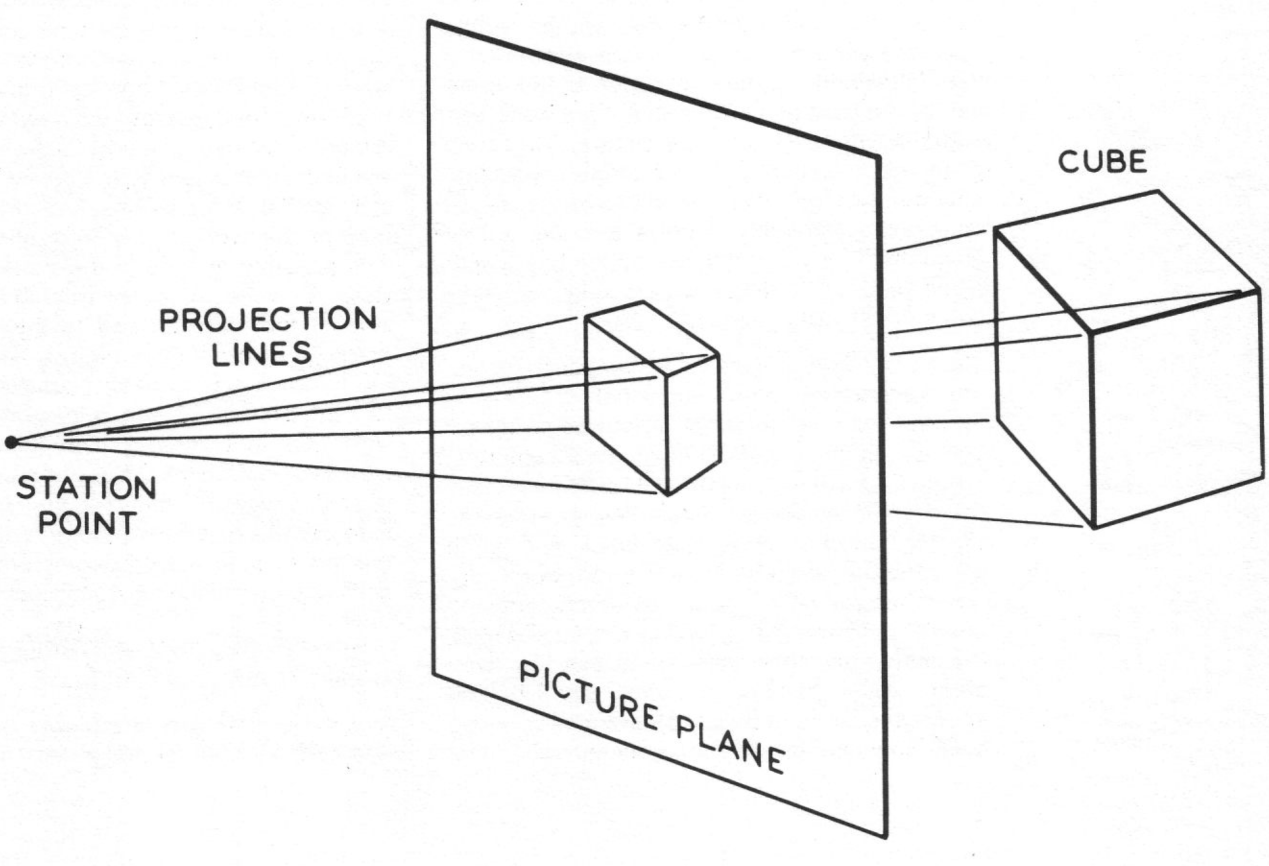
In usual practice a perspective drawing is produced by choosing a point (representing the eye and formally called the station point) from which the object is viewed. A picture plane is then inserted between the object and the station point, and projection lines are drawn from the object to the station point. The points of intersection of these projection lines with the picture plane are joined together to produce the perspective drawing shown below. In this way a three-dimensional object is projected onto a two-dimensional plane.

Two perspective drawings obtained from two picture planes and two station points produce what is called a *stereographic drawing*. Since the two station points are located in slightly different positions, two slightly different perspectives are obtained. When viewed stereoptically, these minute differences in the pictures presented to the left and right eyes are translated by the brain into a depth effect. The computer is unable to physically draw lines from the object to the station points, and therefore an analytic treatment of the projection technique is required. The derivation of these projection formulae is straightforward.

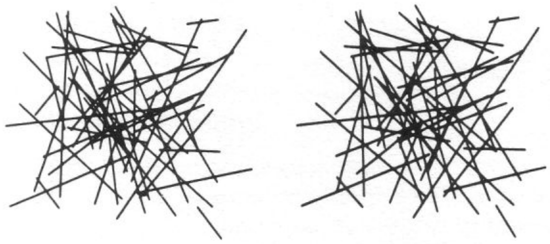
The equations resulting from the analytic geometry approach to perspective projection have been incorporated into a computer program for generating perspective pairs which when viewed stereoptically produce a depth illusion. The three-dimensional object is specified by the rectangular coordinates of its points, i.e., each point in the object is determined by three numerical coordinates rather than just two as for two-dimensional objects. The program then calculates the projections of these points for both the left and the right perspectives. The projections are two-dimensional, and therefore only two coordinates are required. The microfilm plotter draws the left and right perspectives on 35-mm microfilm which can be viewed in a standard stereoscope.

In looking at the left and right perspective drawings shown adjacent to each other, a depth effect can be obtained by decoupling one's eyes sufficiently to produce double images. This task is made easier by first gazing beyond the page at some distant wall or object and then dropping the eyes without refocusing back to the page. Sometimes a piece of paper placed between and perpendicular to the two perspectives may be

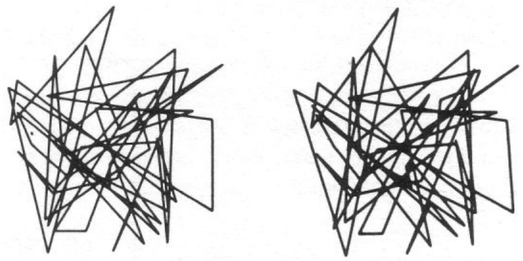
Perspective projection of a three-dimensional cube onto a two-dimensional picture plane.



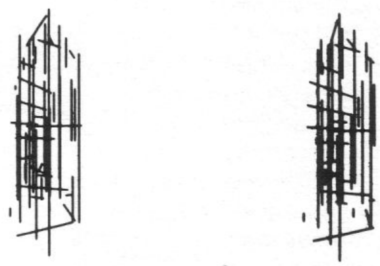
Three-dimensional picture pair of a random structure consisting of 50 lines falling at random within a cube. (figure a)



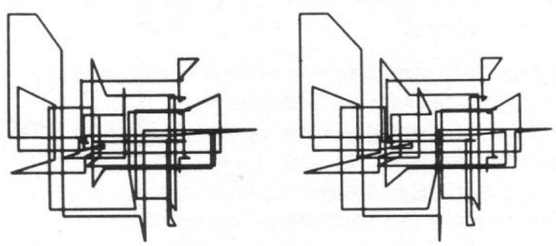
Three-dimensional picture pair of a random line connecting 50 points chosen to fall randomly within a cube. (figure b)



Three-dimensional picture pair of structure consisting of 30 random-length vertical lines and 20 random-length horizontal lines. (figure c)

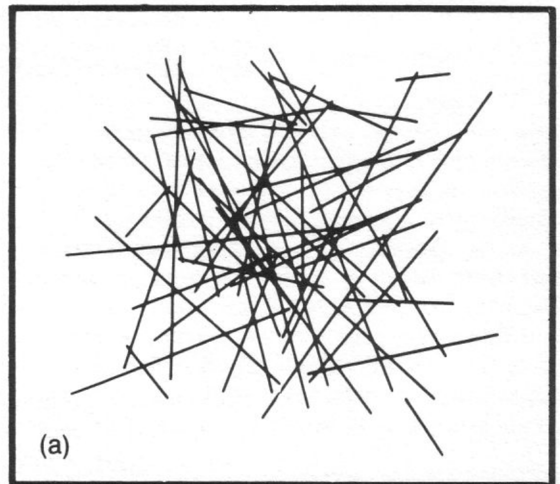


Three-dimensional picture pair of 85 mutually perpendicular connected line segments. (figure d)

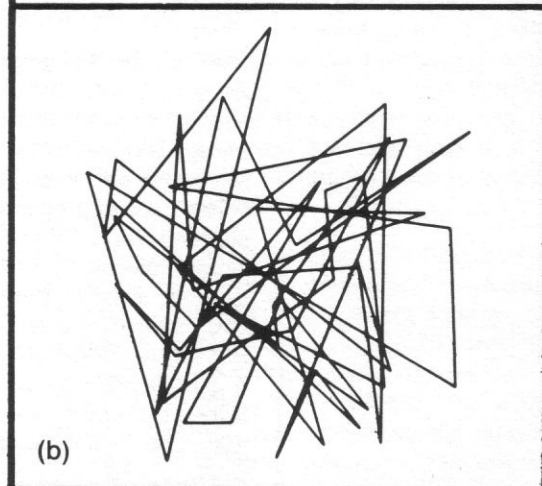


helpful. The desired result is to obtain three separate images. The center image will look solid and will become clear if one looks at it long enough.

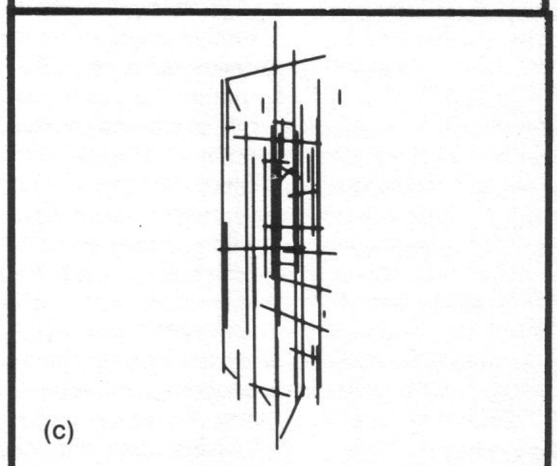
Four computer-generated three-dimensional pictures and their two-dimensional counterparts were used as stimuli in a preliminary subjective test to determine aesthetic judgments for three- and two-dimensional pictures shown above.
The first picture is comprised of 50 lines determined by end points chosen at random to fall within a cube so that all locations within the cube are equally probable. The second consists of 49 line segments connecting sequentially 50 random points once again chosen to fall within a cube. Thirty random-length vertical lines and twenty random-length horizontal lines with end points chosen randomly within a cuboid are shown in the third picture. The object shown in the fourth picture consists of 85 line segments connecting 86 points randomly chosen within a cube. The constraint has been added that only one coordinate can change from point to point, with the result that the lines are always perpendicular to each other.



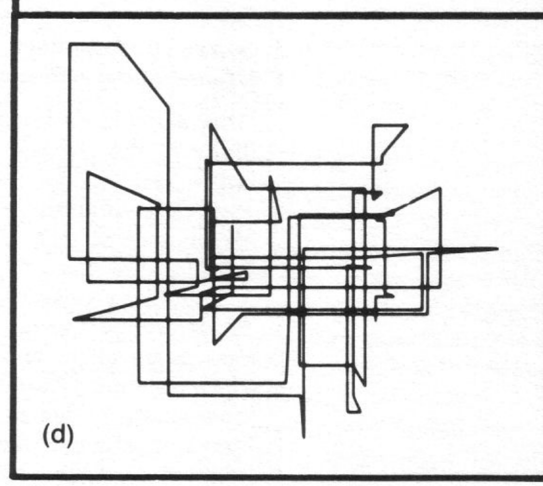
(a)



(b)



(c)

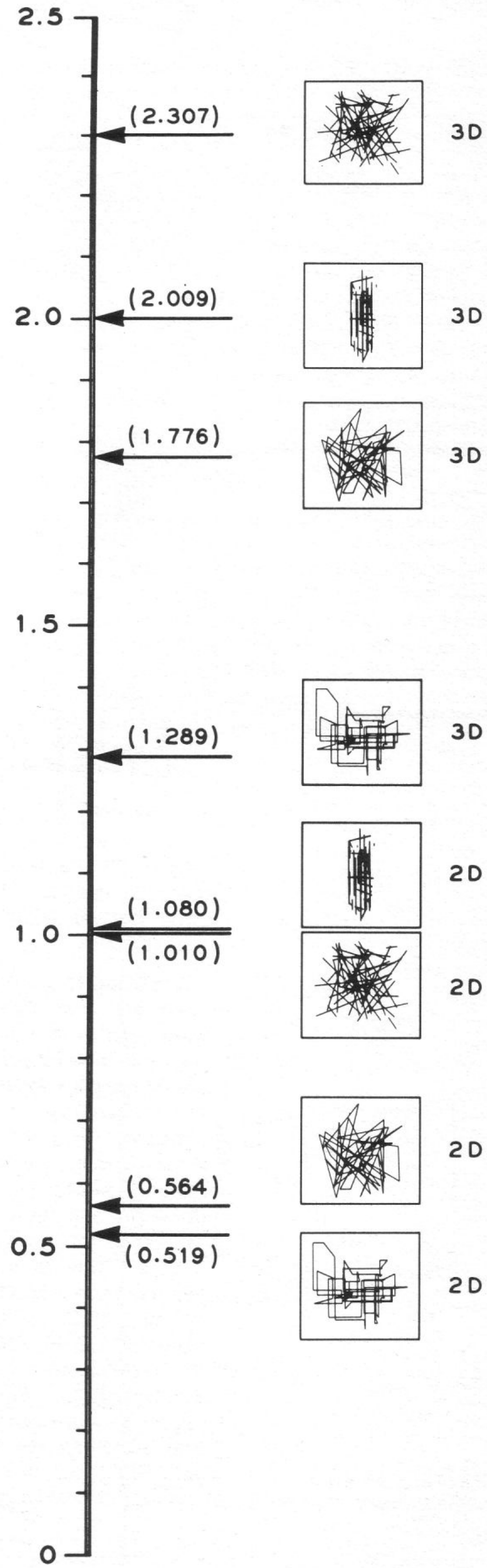


(d)

Far right: Two-dimensional versions of structures a, b, c and d.

Final subjective ordering of three- and two-dimensional patterns. The numerical rating scale is a measure of the psychological distance between the patterns.

EQUAL PREFERENCE SCALE



The computer produced three-dimensional pairs for the four objects in which both perspctives were identical; when viewed stereoptically such pairs look two-dimensional.

Also two-dimensional versions of the four objects were produced by computer. The result was that eight pictures were now available: three- and two-dimensional pairs of four different three-dimensional objects. The eight stimuli were presented to 52 subjects (9 art students and 43 technically-trained people) whose task was to order the stimuli according to their artistic preferences for the pictures. Each subject was tested individually and viewed the eight stimuli in a hand-held stereoscope. A simple test was administered to each subject to be certain that he could fuse the stereo pairs and obtain a satisfactory depth effect.

A statistical analysis of the data was used to obtain final ordering of the pictures and the associated rating values as shown at right. The numerical rating values are a measure of the psychological distance between the different stimuli. The results of this experiment indicate that the three-dimensional version of each pattern was always preferred over its two-dimensional counterpart as shown right. Also, the lowest rated three-dimensional pattern was preferred over the highest rated two-dimensional pattern. In all cases, the disconnected line patterns were preferred over the connected line patterns. All this would seem to indicate that the 52 subjects participating in the test preferred three-dimensional "disconnected" line objects. This is quite surprising since it was expected that most people would prefer the more orderly connected patterns.

This experiment may have important ramifications in the general area of aesthetics. Once again, as in the Mondrian test, computer-generated patterns were used as stimuli in a psychological test, but this time some of the stimuli were three dimensional. This means that it should be possible to give meaningful tests to determine the difference in aesthetic judgments between two- and three-dimensional presentations. Computer-generated pictures are completely objective and can be constructed from mathematically-controlled mixtures of order and random elements.

However it is quite obvious that considerably more testing will be required to answer adequately the aesthetic questions raised by the previous experiment.

As an example of the ease with which order can be united with randomness, the computer was programmed to generate its version of Richard Lippold's sculpture "Orpheus and Apollo" which hangs in the lobby of New York City's Philharmonic Hall and is shown below. This work consists of long flat plates of brass that have been hung from the ceiling by thin wires. For all practical purposes, the plates can be represented by single straight lines. When Lippold's work is

Computer sculpture combining random scattering of lines about specified, but never drawn, trend lines.

so visualized, it becomes possible to describe the sculpture in terms of imaginary trend lines about which the bars have been placed. The computer approach was to specify each trend line by giving the coordinates of its end points, and the computer then distributed lines randomly about this trend line. These lines were random distances from the trend line and also had random angular positions in space. In the result, a total of six such trend lines is used (shown above).

The three-dimensional projection program has the flexibility of specifying any viewing position. In this way it is possible to obtain views of a computer sculpture from any specified position without the necessity of actually constructing the sculpture. This facility should be valuable in visualizing complicated sculptures before the expense of final construction.

The previous comments about the desirability of display consoles for the user to sit at and see the picture as it is being made by the computer are equally applicable to three-dimensional pictures. Here, the left and right images could appear adjacent to each other, and some type of simple optical device could be used to achieve a depth effect. Facilities would also be desirable for shaping three-dimensional objects and instantaneously seeing the results. This might be accomplished with some form of electronic "gloves" whose positions would be sensed by the computer.

THREE-DIMENSIONAL MOVIES (COMPUTER KINETIC SCULPTURE)

Three-dimensional pictures are definitely static, and some form of dynamic display was therefore investigated next. The result was an extension of the three-dimensional program to produce three-dimensional movies. Almost all of the static three-dimensional pictures could have as easily been made out of wire using conventional sculpture techniques. The introduction of randomness and the absence of supporting elements are two advantages of the computer sculpture but are really of little importance.

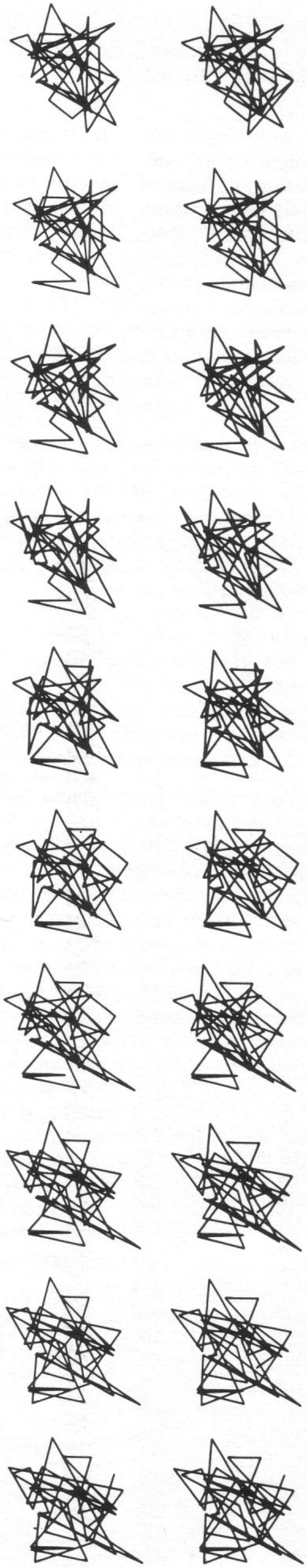
But if the computer can produce a single three-dimensional picture, then it also can produce a series of three-dimensional pictures to make a three-dimensional movie. Now the static character of the computer sculpture is gone and in its place are the almost limitless possibilities of three-dimensional movement and shape transitions.

The three-dimensional projection programs were modified so that three-dimensional movies could be easily specified. The primary change in the programs was one which allowed only those portions of the picture that differed from the previous picture to be projected. This eliminated unnecessary calculations by the computer and resulted in faster programs. The microfilm plotter was fitted with a 16-mm camera so that the developed film could be viewed with a standard 16-mm movie projector.

The procedure for generating a three-dimensional movie with the computer is to mathematically specify the three-dimensional coordinates of the points in a line representation of the desired object. The projection program then computes the corresponding points for the left and right perspectives and generates instructions for the microfilm plotter to draw a single frame of the movie. The next position or shape of the object is mathematically specified, the perspective points are computed, and instructions are generated for producing another frame of the movie. This process is repeated until the movie has been completed on a frame-by-frame basis. The whole procedure is somewhat similar to the conventional animation process.

Some problems existed at first in obtaining a stereoscopic process suitable for viewing the films. Fortunately, this was solved with a commercially available prism device for separately polarizing and then superimposing the left and right perspectives when projected on a screen. The viewer wears polarized glasses so that each eye sees only a single image. The two perspectives might also be projected alongside each other, but the observer would then have to look crosseyed at the two images in order to obtain a fused center image.

Selected frames from a computer-generated three-dimensional movie of a randomly changing random object or a new form of "kinetic sculpture."



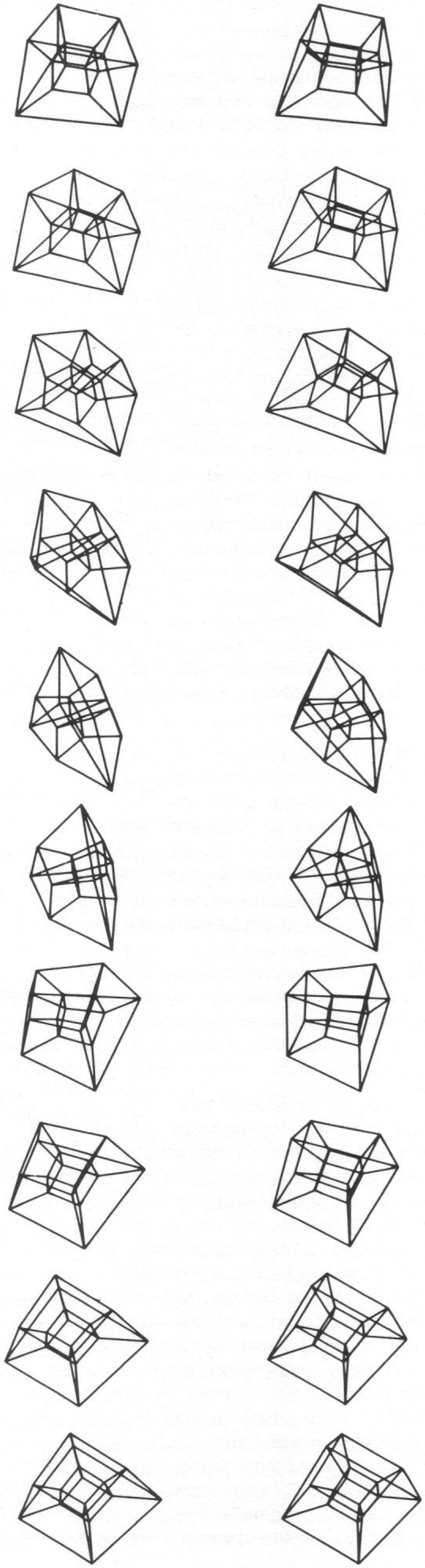
In the first attempt at a computer-generated three-dimensional movie the object consisted of 39 line segments sequentially connecting 40 points picked at random (uniform density) to fall within a cube as shown at left. At randomly chosen times one of the points is given a new random position within the cube, and the two lines attached to it are instantaneously twisted to new orientations. This is truly a "kinetic sculpture". Finally the computer has produced an artistic effect unattainable by any other means. There have been attempts previously by artists to paint by hand three-dimensional films, but attaining the detail of the computer film would be extremely time consuming and would probably require a lifetime.

The three-dimensional projections were introduced as an added improvement on two-dimensional pictures. In a way, a fourth dimension in the form of time was added in the three-dimensional movies. Now a fifth dimension is added as a computer-generated three-dimensional movie of four-spatial-dimensional objects. The world as we know it has three spatial dimensions. However, it is quite possible to speak mathematically of four and even higher spatial dimensions. Indeed, many scientific and technical data have four or even more dimensions, and cannot be fully plotted or represented in three-dimensional space. However, dimensions higher than three can have no physical significance to us since we are only able to visualize three spatial dimensions. In the same way that a three-dimensional object is specified by three coordinates, a four-dimensional object is specified by four coordinates. In other words, if a line representation is made of the four-dimensional object, then each point is located in four-dimensional space by four numbers.

Although it is possible mathematically to specify four-dimensional objects, it is impossible to see such an object. However, it is possible to produce a three-dimensional projection of a four-dimensional object. This is accomplished by mathematical formulae similar to those used to produce a two-dimensional perspective of a three-dimensional object. The four-dimensional object can be rotated in four-dimensional space by mathematical operations on the coordinates representing its points. The resulting orientation can then be projected down to three dimensions. All these operations are mathematical and hence can be performed by the computer. The result is a three-dimensional movie of the three-dimension conical projection of a rotating four-dimensional object.

The four-dimensional analogue of the cube is called a four-dimensional hypercube. A computer-generated movie of a rotating hypercube was made (shown right). The rotations which involve the fourth dimension result in the object appearing to turn inside out. The

Selected frames from movie of the three-dimensional projection of a rotating four-dimensional hypercube.



motion itself is very intriguing and, although very complicated, immediately implies a sophisticated generating process.

The programs and mathematical techniques for four-dimensional projections and rotations are quite general and can easily be extended to even higher dimensions.

Three-dimensional movie production would be greatly facilitated by having immediate display of the movie as it is being generated by the computer. As in the case of three-dimensional static pictures, some form of optical device could be used for viewing the two perspectives to obtain a depth effect. The user could control different parameters and see their immediate effects on the movie. As an example, the angles of rotation of the four-dimensional object might be controlled by knobs that the user could turn while seeing the effects of the rotation. Such devices will very soon be available, and such experiments in the presentation of objects and data will undoubtedly be attempted.

COMPUTER CHOREOGRAPHY

Stick figure motion on a stage can be shown by a sequence of movie frames. Each stick figure consists of a single line for the body, a single-line shoulder, and single-line arms. The arm positions are completely variable, and the size of each body element can be individually specified. The whole stick figure can be rotated to any specified angle and located at any position on the stage. In one particular example, six figures were used(illustrated page78). Three move their arms uniformly up and down. The stage motion is random (any position is equally likely), but only one coordinate changes each time so that the motion is always parallel to the edges of the stage. The motion from position to position is at a uniform rate that is individually specified for each figure. At random times all stage motion ceases, and the three figures with the moving arms make one complete turn. The effect is reminiscent of the motion of atoms in a gas. The stage motion then continues. The three-dimensional movie was generated on the computer using the method described in the previous section.

This example demonstrates the possibilities for introducing controlled combinations of order and randomness. The stick figures used were quite simple but did, however, require many detailed specifications. The extension to more elaborate stick figures would require new programming techniques in which movement of the limbs is combined into basic movements which might then be combined on an even higher level. In this way the most complicated dance motion could be specified as a combination of relatively simple movements. Human movement is extremely complex and obtaining the equations for as simple a motion as walking would be formidable. A better attack on this problem

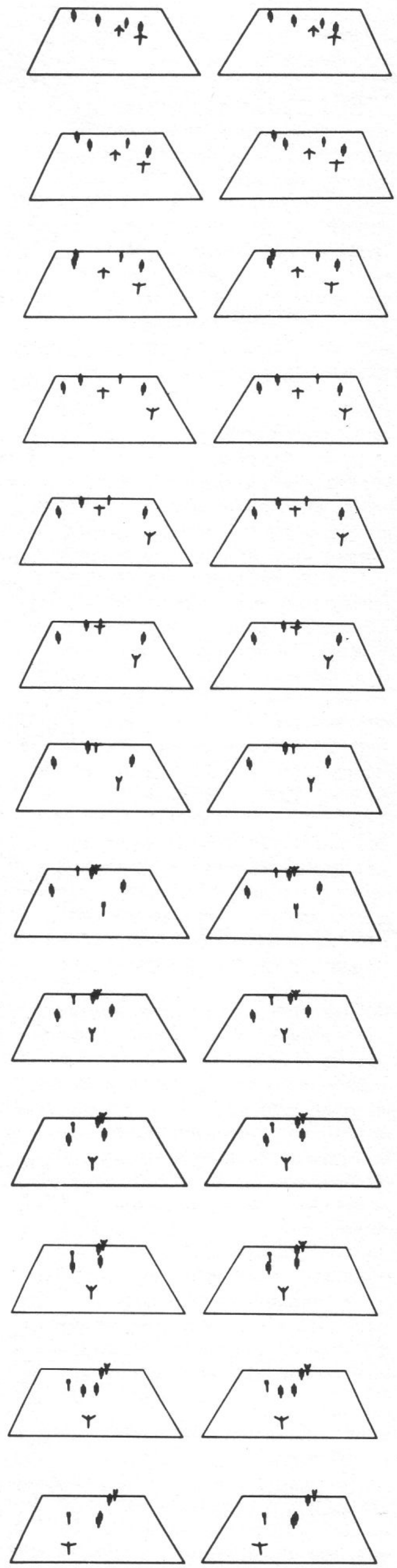
Far right: Selected frames from a computer-generated three-dimensional movie of stick figure representation of human motion on a stage or a form of "computer choreography."

might be for the computer itself to analyze human motion using devices that have just become available for converting pictorial data into machine digestible data. A library of basic movements could be built up within the computer, and particular movements could be then put together at will.

The dance world is currently in the midst of problems concerning dance notation. The problem might be likened to a composer who requires a full symphony orchestra at his disposal in order to try different orchestrations as he thinks of them. When the final musical work has been obtained, each musician has to remember his own part since there has been no method of notation. In dance, the choreographer requires the whole ballet corps to be present for him to create the ballet, and when finished no notation is available. Some dance notations do exist, but they are neither understood nor used by most choreographers. Also, most of the notations are so extremely elaborate that transcription is very time consuming. Thus, dance notation is understandably little used although everyone recognizes the need for it. Film, incidentally, is not the answer, since this would only be a method of recording one ballet corps' particular interpretation of a work. Perhaps computers might offer a way out of the present dilemma.

One solution to the notation problem would be to use a filmed version of the ballet as input to a computer. Three different camera positions could be used simultaneously, and the dancers would wear light sources so that only light would be obtained on the final films. The films would then be analyzed by the computer and converted into motion patterns. These patterns could then be translated into any desired dance notation and drawn on microfilm or other suitable hard copy. However, this approach would require some sophisticated technology. Instead, changes in the present choreographic creative process might in the long run be better.

Many choreographers require the actual presence of the ballet corps to immediately perform the dance steps as the choreographer creates them. This is very wasteful of the dancers' time which could be better spent in rehearsing and further training. In a way, it is also restrictive because the resulting ballet is strongly related to one particular corps of dancers. As an alternative to this present approach, one can assume that the choreographer has a digital computer with some form of real-time visual display at his disposal. Instead of using the ballet corps as his choreographic instrument, the choreographer could interact with the computer during the creative process. Stick figure representations of the dancers could appear in some form of three-dimensional display on the face of an electronic display tube. The choreographer, by manipulating different buttons on the console,



could control the movement and progress of the ballet. Different dance movements might be stored in the computer's memory and put together at will. Individual movement restrictions for each dancer could even be introduced into the process. At the completion of the ballet, all the movements of all the dancers would be stored within the computer in digital form. These movements could then be automatically translated by the computer into any desired form of dance notation and produced as output. The notated motions would then be given to the dancers. In this manner, the choreographer and dancers would be mutually freed from each other, and each able to individually pursue his own tasks. The notation problems would be automatically solved since the ballet would be built up within the computer and could be easily transcribed by the computer.

These are quite radical proposals for the dance world, and most certainly not every choreographer would wish to create dance with the inanimate machine. However, if ballets were specified in computer form much research into the dance process and movement would be possible. Some of the questions to be answered by such investigations include the differences in style and movement for different choreographers, the relation between music and dance, and the mechanics of human movement.

FUTURE POSSIBILITIES

Presently, equipment for obtaining visual displays while the program is running on the computer is becoming available. With this equipment, an artist-programmer will be able to sit at the console and immediately see the pictorial results of his program. If desired, parameters can be changed, and the new picture will immediately appear on the face of the display tube. In this way, an artist-programmer can interact with the computer through an intermediary display unit. The next experiments in computer art will undoubtedly be concerned with such interactions.

Suppose some artist decides that he would like to try using the computer as an artistic medium. Unfortunately, a number of problems would very rapidly arise. The first would be the availability of a computer with some form of visual output. Computers are presently situated at industrial research laboratories or at universities and are therefore somewhat inaccessible to most artists unless they are associated with a university. But most universities do not even have display equipment. Also, computer programming is a discipline entirely different from anything ever approached in most artists' training. Special programming languages must be learned before one can ever hope to write a program for the computer. All of this easily leads to a pessimistic attitude toward the future of computer art.

Nevertheless, an optimistic approach is reasonable. New computer languages will shortly be developed that will make it very easy to communicate instructions to the machine. Computers with display equipment will also become more readily accessible with the net result that many more people, including artists, will become computer oriented. In time this new artistic medium will be exploited to produce previously unknown effects combining color, depth, motion, and randomness in creative combinations. New information about the psychological aspects of the aesthetic experience will also result.

The new techniques discovered and developed by artists as they use computers could be fed back to the scientists who originated the "art," and thus this could be the beginning of a new era of closer cooperation and better understanding between the arts and science.

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