Reprinted from –
AFIPS – Conference Proceedings, Volume 33

383 National Press Building
Washington, D. C. 20004

Computer animation and the fourth dimension

by A. MICHAEL NOLL

Bell Telephone Laboratories, Incorporated
Murray Hill, New Jersey

INTRODUCTION

Man is a creature restricted to a world of three spatial dimensions in which he is reasonably free to move about at will except for the arbitrary territorial boundaries imposed by different nations. Man also lives in another dimension over which he presently has no control other than to watch its continual forward movement in the mechanical and electrical gadgets he has devised to measure this dimension which he calls time. Many people call time the fourth dimension, but because of its many unique qualities I would rather consider time as a special dimension. Therefore, in this paper the fourth dimension is a fourth purely spatial dimension not to be confused with time.

Since we live in a world of three spatial dimensions, we are unable to visualize a fourth spatial dimension perpendicular to our three spatial dimensions. However, the computer is not bothered by problems of human visualization and is able to deal with objects in four-dimensional space as easily as it performs calculations for three-dimensional objects. Only a fourth number is required by the computer to locate a point in four-dimensional space. But if the computer is to present the results of its calculations with four-dimensional objects to man, then the computer must come down from its digital tower and perform the necessary operations so that man can visualize and possibly obtain some intuitive feel for the results. The technique of perspective projection from four dimensions to three dimensions is particularly helpful here.

In a previous paper ("A Computer Technique for Displaying n-Dimensional Hyperobjects," Communications of the ACM, Vol. 10, No. 8, August 1967, pp. 469–473), I described in general the technique of perspective projection of n-dimensional hyperobjects and also the application of this technique to the production of computer-generated three-dimensional movies of rotating four-dimensional hyperobjects, and in particular, the hypercube. The mathematical details of this previous paper will not be repeated here but a physical interpretation of the results will be explored with particular emphasis on the purely artistic effects obtainable with the technique.

A computer technique for displaying four-dimensional hyperobjects

The computer performs its calculations and other operations under the control of a program as outlined in Figure 1. The computer first reads into its memory the four-dimensional coordinates of each point of the hyperobject. Just as three numbers exactly position a point in three-dimensional space, each set of four numbers exactly positions a point in four-dimensional space. These points usually are the end points of straight lines in four-dimensional space and have been previously punched onto standard punched cards.

Rotation

After the four-dimensional object has been converted to straight lines and the four-dimensional coordinates of the end points of these lines have been read into the

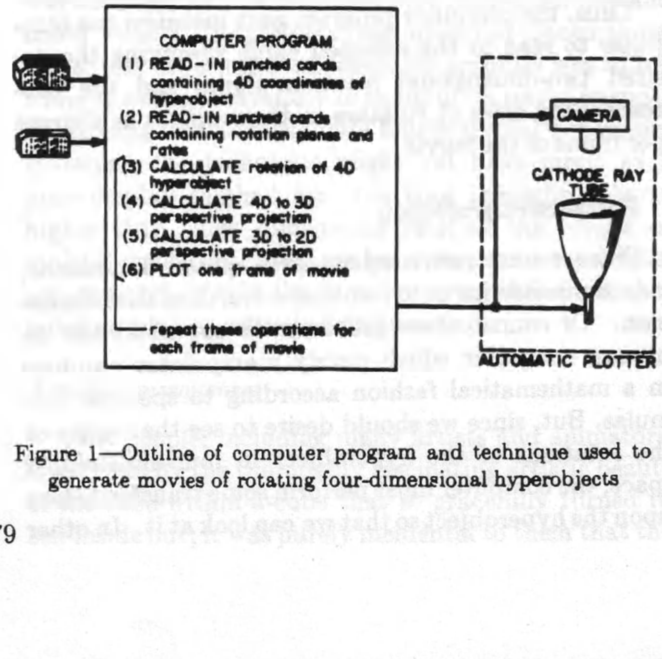

Figure 1—Outline of computer program and technique used to generate movies of rotating four-dimensional hyperobjects

memory of the computer, the object must then be moved in some fashion in the four-dimensional space since a movie of a completely stationary object is quite senseless to most people.

There are many types of motions which can be considered such as shrinking, expanding, translating, or rotating the object in four-dimensional space. I arbitrarily decided to restrict the movie to rotation of the hyperobject in four-dimensional space, but this decision is unfortunately not enough since some thought must be given to exactly what is meant by rotation in a four-dimensional space.

In three dimensions, we usually think of rotation in terms of rotating an object about some specified axis, i.e., the axis of rotation. An alternative approach is to consider the two-dimensional plane that is perpendicular to this axis and define a rotation as a rotation within this two-dimensional plane. If we define a principal rotation to be one which involves a rotation about one of the principal coordinate axes, then the successive application of three rotations about each of these three axes can rotate the three-dimensional object to any specified position. In any single plane perpendicular to one of the principal axes, only two coordinates change as a result of the rotation in that particular plane.

In four dimensions I have decided to extend this three-dimensional concept of rotation. However, in four-dimensional space there are many axes all perpendicular to the same two-dimensional plane so that specifying a rotation about an axis is both intuitively and mathematically meaningless. The solution is to specify the two-dimensional plane in which the rotation is to take place and to completely drop the three-dimensional concept of rotation about an axis. Once again, all of this is somewhat arbitrary since one could define other four-dimensional rotations, but I desired a mathematical definition that was a simple extension of a rotation in three-dimensional space as specified by a two-dimensional plane.

Thus, the computer program next instructs the computer to read in the punched cards specifying the desired two-dimensional rotation planes and the corresponding rates of rotation in these planes as degrees per frame of the movie.

Perspective projection

Since we are restricted to three spatial dimensions, it is impossible for us to visualize a fourth spatial dimension. Of course, these problems do not affect the inhuman computer which purely manipulates numbers in a mathematical fashion according to specified formulas. But, since we should desire to see the results of the rotations of the hyperobject in four-dimensional space, the computer must perform some transformation upon the hyperobject so that we can look at it. In other words, the four-dimensional object has to be projected to three dimensions so that we can see it in our three-dimensional world.

Just as there were many alternatives for various motions, there are many alternatives for the type of projection from four dimensions to three dimensions. One type of projection is parallel projection in which the projection lines from the object to the projection space are all parallel to each other. This is also equivalent to simply eliminating one coordinate in the specification of each four-dimensional point so that one is left with three numbers for each point in the object. However, parallel projection is not usually encountered in our three-dimensional world since our eyes are really a perspective projection system. For this reason, I decided to use perspective projection from four dimensions to three dimensions. The equations for this projection are derived in the previous paper and are a very simple extension of the projection equations for going from three dimensions to two dimensions. The use of perspective projection allows one to retain a three-dimensional feel for what is happening although absolutely no feel for the fourth dimension in terms of rigidity of the four-dimensional object is obtained as will be described later.

Plotting the movie

The three-dimensional projection of the four-dimensional object is finally perspectively projected to two dimensions. These two-dimensional coordinates are used as input to an automatic plotter consisting of a cathode ray tube and a camera positioned to photograph the face of the tube. After each single frame of the movie has been plotted, the computer program instructs the computer to rotate the hyperobject to its next position, to project the hyperobject to three dimensions and then to two dimensions, and finally to plot another frame of the movie on the face of the cathode ray tube. These operations are repeated until the desired number of frames of the movie are obtained.

The hypercube

The four-dimensional hypercube is introduced now because of its importance in understanding the motions that result from rotations in four-dimensional space. The three-dimensional perspective projection of a four-dimensional hypercube looks like a three-dimensional cube within a three-dimensional cube as depicted in Figure 2. Some intuitive insight can be gained by considering the two-dimensional perspective projection of a three-dimensional cube, as depicted in Figure 3, which looks like a square within a square. What is happening here is that the face of the cube closest to the projection

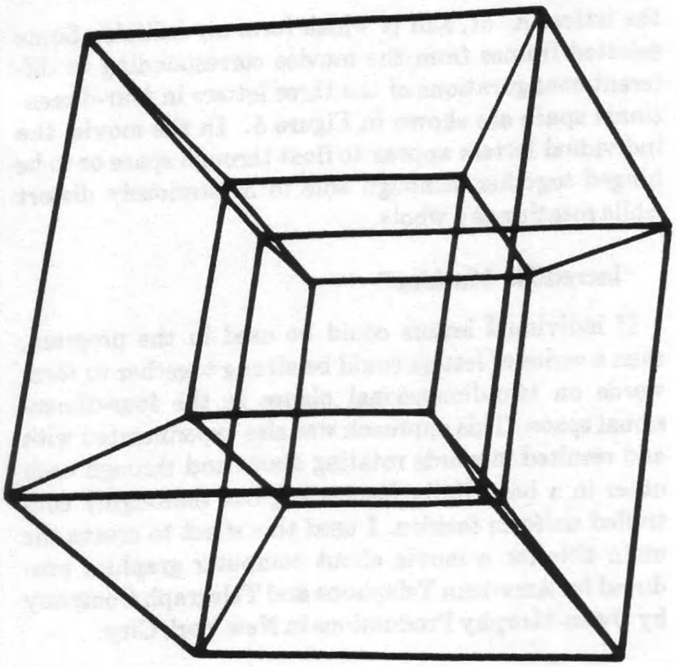

Figure 2—The three-dimensional perspective projection of a four-dimensional hypercube is a cube-within-a-cube with the outer cube corresponding to the face of the hypercube which is closest to the projection space

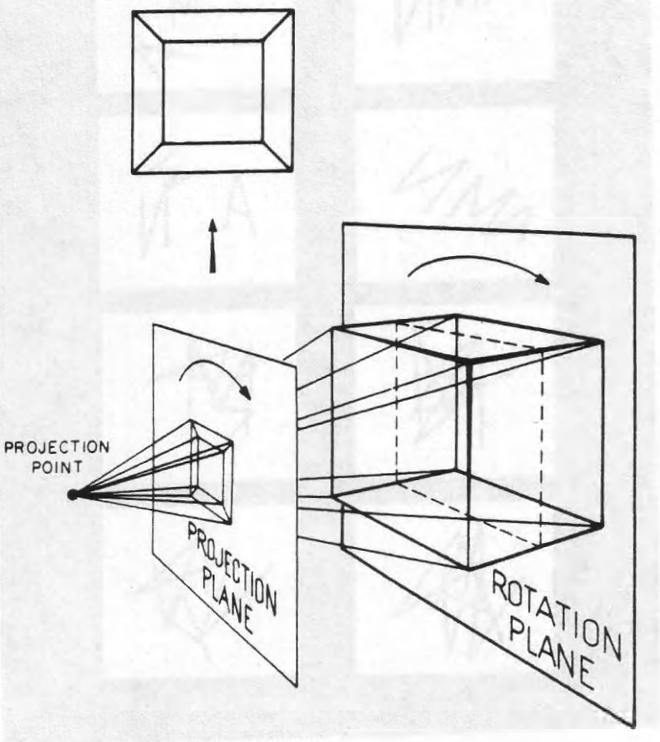

Figure 3—The perspective projection of a three-dimensional cube with one face parallel to the projection plane is a square-within-a-square. If the rotation plane is perpendicular to the axis along which the cube is being viewed, then the square-within-a-square simply turns as a whole

plane appears largest while the face farthest away is smallest. If the closest face is paralled to the projection plane, then its perspective projection is undistorted except for its overall size. Thus, the perspective projection of a three-dimensional cube is actually an undistorted two-dimensional face within an undistorted two-dimensional face. Since the face of a four-dimensional hypercube is a three-dimensional cube, the perspective projection of the hypercube with one face parallel to the projection space is a cube within a cube.

When the hypercube is rotated, some very intriguing distortions occur in the three-dimensional perspective projection depending upon the choice of rotation plane. If the two-dimensional rotation plane is perpendicular to the axis along which the hypercube is being viewed, then the cube-within-a-cube turns as a whole. If the rotation plane is not perpendicular to the viewing axis, then the inner cube enlarges until it becomes the outer cube while the outer cube shrinks until it becomes the inner cube. This turning-inside-out motion is to be expected since as the hypercube turns one face comes closest to the projection plane and therefore appears largest. All of this is similar to the three-dimensional cube that rotates in a plane which is not parallel to the projection plane except that the faces of the three-dimensional cube are squares which distort and change their size as different faces come closer or recede from the projection plane as shown in Figure 4.

The four-dimensional cube is of course completely rigid while it is being rotated, and the turning-inside-out motion is purely a result of the perspective projection from four dimensions to three dimensions. It was hoped that one might be able to obtain some feel or visualization for the rigid four-dimensional hypercube by observing the three-dimensional perspective projection but no such feel could be obtained. On the contrary, only a cube-within-a-cube turning inside out was observed as if its members were made of rubber and could stretch to perform the observed contortions. Thus, this experiment in computer graphics was something of a complete failure in terms of its stated purpose of assisting man to visualize a fourth spatial dimension. However, the technique might yet have merit as a pure display method for observing scientific data in higher than three dimensions such as the results of multi-dimensional factor analyses of experimental data, and research in this direction involving real-time computer graphics is actively being pursued.

Artistic consequences

Most people, including many artists and animators, were awed by the thoroughly fascinating artistic beauty of the cube-within-a-cube that so gracefully turned itself inside out; it was purely incidental to them that the

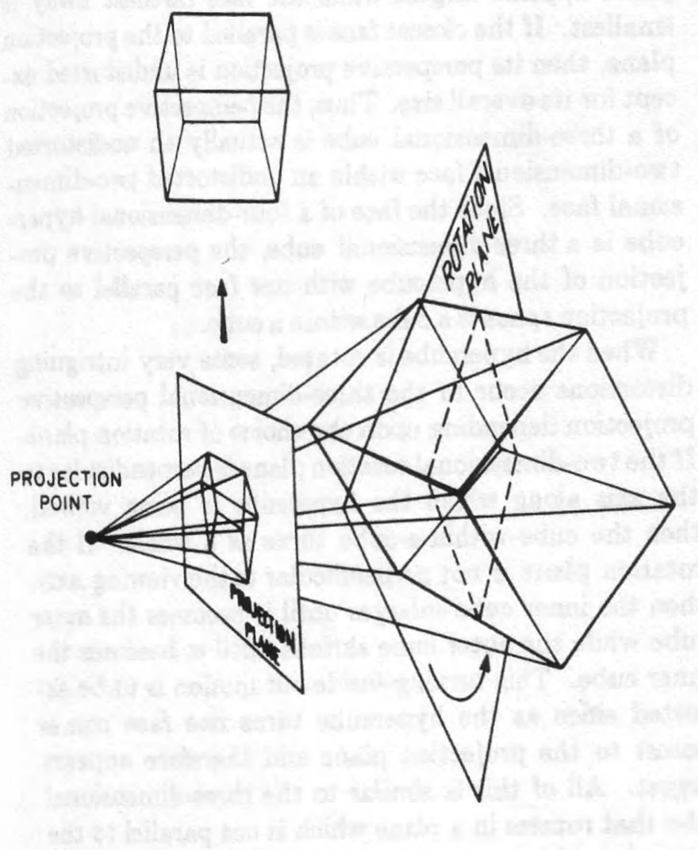

Figure 4—If the rotation plane is not perpendicular to the viewing axis, then the faces appear as distorted squares which change both their size and shape as the cube rotates

movie was produced by a digital computer or that the cube-within-a-cube was the perspective projection of a four-dimensional hypercube. Some further investigation and exploitation of the technique for its artistic consequences alone was quite definitely indicated by all the enthusiasm generated by the movie. The remainder of this paper describes these artistic explorations as far as it is possible to verbalize matters involving aesthetic concepts.

Four-dimensional letters

The computer program for producing the movie of the hypercube was general purpose in the sense that any set of four-dimensional points could be used as input. In my mind, I visualized letters rotating and moving through each other in an extremely graceful yet obviously thoroughly controlled manner similar to the motion of the hypercube. To investigate this effect, I used the four-dimensional coordinates of points specifying the end points of straight lines as input to the program. These straight lines formed individual letters which I could easily in effect place on different two-dimensional planes in the four-dimensional space. Conceptually, this is identical to placing the letters on the two-dimensional planes which make up the hypercube. I chose the letters A, M, and N which form my initials. Some selected frames from the movies corresponding to different configurations of the three letters in four-dimensional space are shown in Figure 5. In the movie, the individual letters appear to float through space or to be hinged together although able to mysteriously distort while rotating as a whole.

"Incredible Machine"

If individual letters could be used in the program, then a series of letters could be strung together to form words on two-dimensional planes in the four-dimensional space. This approach was also experimented with and resulted in words rotating about and through each other in a beautifully fascinating but thoroughly controlled uniform fashion. I used this effect to create the main title for a movie about computer graphics produced for American Telephone and Telegraph Company by Owen-Murphy Productions in New York City.

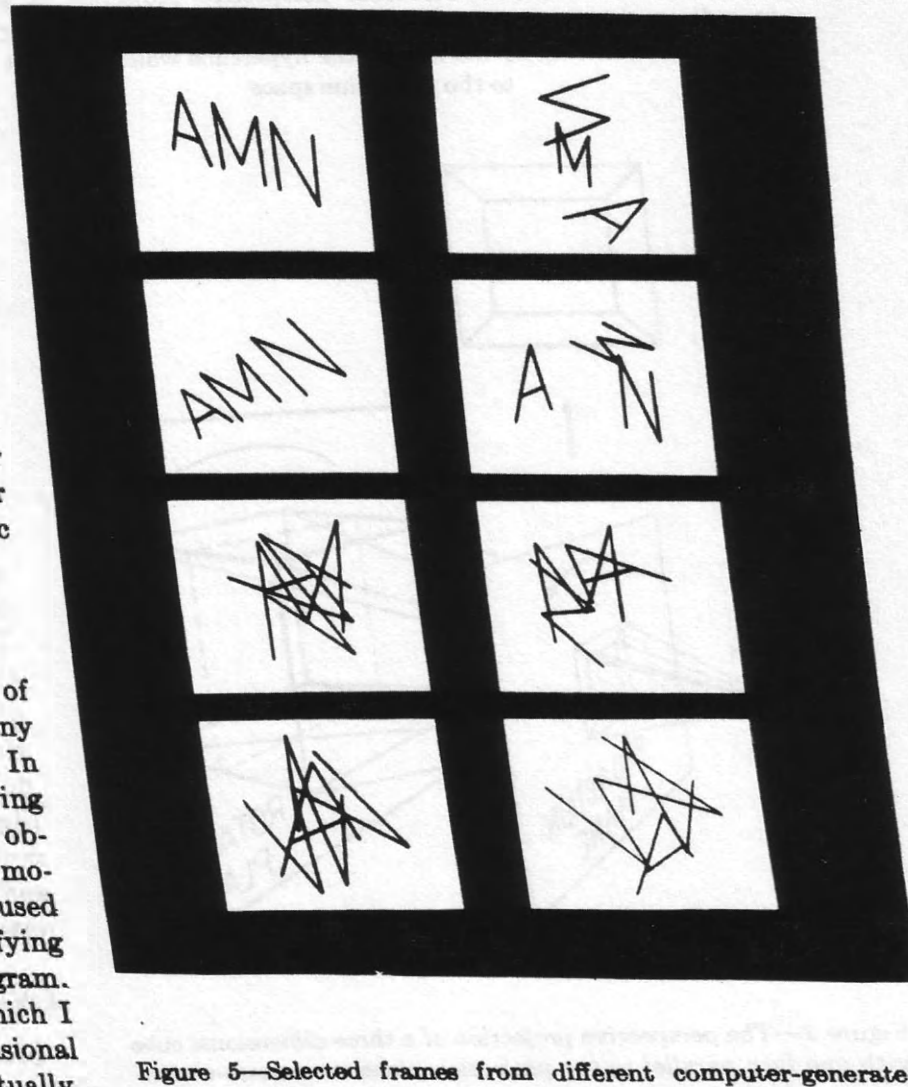

Figure 5—Selected frames from different computer-generated movies of different configurations or placements of the letters A, M, and N on different two-dimensional planes in four-dimensional space

The individual letters of the title of this movie "Incredible Machine" were converted to straight lines connecting points whose two-dimensional coordinates were then obtained by plotting the letters of each word on graph paper. The two words were then in effect placed upon two different two-dimensional planes in four-dimensional space, and the four-dimensional points of the end points of the lines forming the letters were punched into cards for input to the computer. A number of different configurations of the two words in four-dimensional space were then appraised for their artistic merit by my judgment of the effectiveness of the computer-generated movies of the rotating words. A single configuration was finally chosen, and fairly long movie of the rotating two words was produced. From this movie, a single frame was chosen in which the two words both were individually legible and also formed an aesthetically pleasing overall form.

The final title movie opens with a zoom-in on the rotating words. The zoom has a linearly-decreasing acceleration so that transition at the end of the zoom to the rotating portion is smooth. The rotating portion continues until the previously-mentioned frame is reached whereupon all motion ceases for a very short while. This freeze is followed by another zoom-in which was done optically rather than by the computer since this was easier than programming the computer to determine which lines exceeded the picture area. The final title movie was optically superimposed over the opening scene of a man working at a graphical display console. Selected frames from the title movie including the freeze frame are shown in Figure 6.

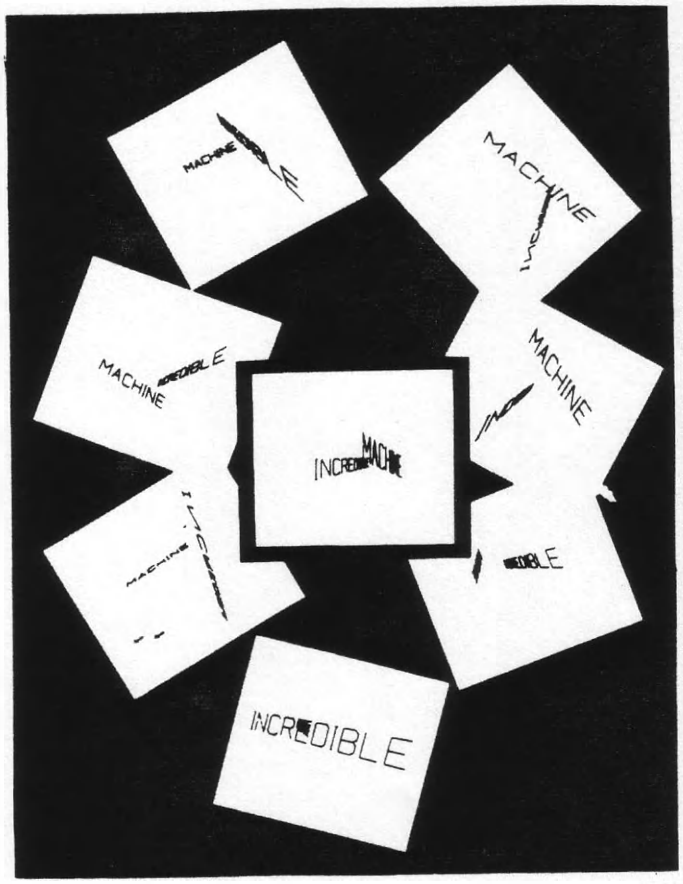

Figure 6—Selected frames from the computer-generated main title of a movie about computer graphics entitled "Incredible Machine"

CONCLUSION

A question arises in some people's minds about the artistic merit of purely scientific techniques which are applied to artistic purposes. I would like to think that the end result, no matter through what medium it was produced, should be judged for its own artistic merit. The fact that these movies were produced by a digital computer performing all sorts of mathematical operations on four-dimensional data should be incidental to the artistic effects thereby achieved. However, I do feel that these techniques involving the new technology, and in particular computers, will only be exploited fully for artistic purposes when the artist who has dedicated his life to artistic explorations learns to use these new tools as new artistic media. I am an engineer and my artistic ideas are somewhat conservative. But, I am quite excited by the prospects for the new artistic effects and beauty which will surely result from creative collaboration between artists and the computer.